

# Tax-Managed Factor Strategies

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Factor tilts are attractive to taxable investors, but the turnover required to maintain a factor tilt can erode tax alpha. We measure this effect in seven factor tilts in the US and global markets. Over the period 1995-2017, the performance hit was modest in beta 1 strategies although more substantial in low-risk strategies. Here are the highlights.

- Median estate/donation tax alpha for beta 1 factor tilts was between 1.7% and 2.2%. For low-risk strategies, the range was between -0.5% and 1.8%.
- The factor alpha required to overcome tax alpha erosion (hurdle rate) was between 0.44% and 2.66%. The highest hurdle rates were for the low-risk strategies.
- Loss harvesting added no more than 0.55% to the tracking error of a factor tilt.

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#### For Professional Investor Use Only

References to returns, risks, performance, tracking error, and other such characteristics describing portfolios in this paper are based on hypothetical analysis techniques (also known as back-testing) and do not represent actual portfolios. Since returns included herein are hypothetical and based on back-testing, it is important to note that they are for illustrative purposes only. Past performance, whether illustrative or actual, is not a guarantee of future performance. Please refer to the important disclosures within and at the end of this paper.

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## Introduction

For a taxable investor, any deviation from a passive strategy requires conviction that the alpha is big enough to outpace the tax drag from turnover. The issue is crystallized in a long-only value tilt, where factor alpha depends on price appreciation of stocks with low valuations. Unchecked, the strategy sheds winners once they become too expensive, which may lead to tax payments that erode the tilt's performance. What is the tax alpha reduction of various factor-tilted strategies versus a simple index-tracking loss-harvesting strategy? Can investors still enjoy a loss-harvesting benefit while attempting to capture factor premiums? We reframe these questions in terms of a hurdle rate, which can inform a taxable investor's decision about which strategy to select. Trades that maintain a factor tilt can lead to the realization of gains, but the degree to which this erodes performance depends on the tilt. Each factor tilt has its own hurdle rate—the outperformance required to compensate for the sacrifice in tax alpha that comes from choosing the tilt over a tax-managed indexing strategy. Will the alpha delivered by a factor tilt exceed its hurdle rate? The answer must be yes in order for the tilt to be a rational choice for a taxable investor.

In this note, we look at the tax-loss harvesting potential of Aperio's tax-managed factor tilts, which come in two varieties. Beta 1 strategies seek to outperform a benchmark by tilting toward factors while maintaining reasonable levels of tracking error. Low-risk strategies seek to reduce total portfolio risk while outperforming on a risk-adjusted basis. Since we are looking at popular strategies that have been successful in the past, the pre-tax results are good: these strategies have generally beaten their benchmarks. To understand the sources of the outperformance and estimate hurdle rates, we decompose after-tax active return, the difference in return between a factor tilt and its benchmark after tax, into a component due to the factors and a component due to loss harvesting.

Loss harvesting amplifies the substantial period sensitivity displayed by tax-indifferent investment strategies, so performance over a single period doesn't tell us very much.<sup>1</sup> To address this, we look at the range of outcomes of tax-managed tilts launched at frequent start dates. Loss harvesting also leads to horizon dependence, with tax alpha generally higher and tracking error generally lower at shorter horizons.<sup>2</sup> In this article, we focus on a horizon of 10 years, which is a typical target for a high-net-worth investor.

## Study Outline and Performance Metrics

We launch 10-year factor-tilted strategies on a quarterly basis over the period June 1995 to December 2017, resulting in 50 different runs for each strategy. Our menu of factor tilts is in Table 1, along with the universes of securities from which we construct portfolios and the indexes against which we measure performance.<sup>3</sup> Strategies are rebalanced monthly, and we

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<sup>1</sup> The term-dependence of tax alpha in an indexing strategy in the US market is explored in Goldberg et al. (2017).

<sup>2</sup> Our historical analysis is courtesy of ATBAT, Aperio's After-Tax Back-Testing Analysis Tool.

<sup>3</sup> "Index" translates to "Market" in Barra documentation.

assume a round-trip trading cost of 12 basis points. We assume strategies are taxed at the highest federal rates, which are listed in Table 2. Implementation details are in Appendix B.<sup>4</sup>

	US		Global	
	Universe	Index	Universe	Index
Base	S&P 500	S&P 500	MSCI ACWI	
Value	R1000	S&P 500		
Value Momentum	R3000	R3000		
Small Value	R3000	R3000		
Multi-Factor	R1000	R1000		
Quality Light	R1000	S&P 500		
Quality	R1000	S&P 500		
Min Vol Value	R3000	S&P 500		

**Table 1:** Menu of factor tilts.

Tax Rate	
Short-Term	40.8%
Long-Term	23.8%

**Table 2:** Assumed tax rates are at the highest federal level as of January 2018.

At each monthly rebalancing, we construct factor tilts in US and global markets by minimizing forecast tracking error to the appropriate benchmark, subject to constraints. These include limits on factor exposures as well as restrictions on positions. In tax-managed strategies, we harvest losses as well, and the relative importance of the tilt constraints, risk management, and tax efficiency are configurable. Strategy construction details are in Appendix B.

Our comprehensive performance metric is *after-tax active return*, the difference between portfolio return and benchmark return after tax. In a tax-managed factor tilt, after-tax active return comes from both factor performance and loss harvesting, so we break after-tax active return into a component due to factors and a component due to tax.

To implement this decomposition, we introduce a tax-indifferent factor tilt of the kind that might be constructed for a pension, endowment, or any other tax-exempt entity. *Factor alpha* is the active return of a tax-indifferent tilt, the difference between a tax-indifferent tilt and its benchmark.

*Tax alpha* can be further broken down into two components. The *portfolio tax benefit* is the value to the investor of realized losses less realized gains, and its generation is the main

<sup>4</sup> Strategy settings used for simulation and live performance differ since the latter is customized to individual investors.

objective of tax-loss harvesting. Since it is not possible to harvest losses without deviating from a benchmark or a pre-tax strategy, loss-harvesting always leads to *tracking return*, the deviation between tax-managed and tax-indifferent versions of the strategy pre-tax.

$$\text{After-Tax Active Return} = \text{Factor Alpha} + \underbrace{\text{Tax Alpha}}_{\substack{\text{Portfolio Tax Benefit} \\ + \\ \text{Tracking Return}}}$$

After-tax performance calculations come in two varieties. In the estate/donation disposition, wealth is either bequeathed or donated, and taxes are never paid. In the liquidation disposition, payment of taxes is delayed. We consider both dispositions in the analysis below.

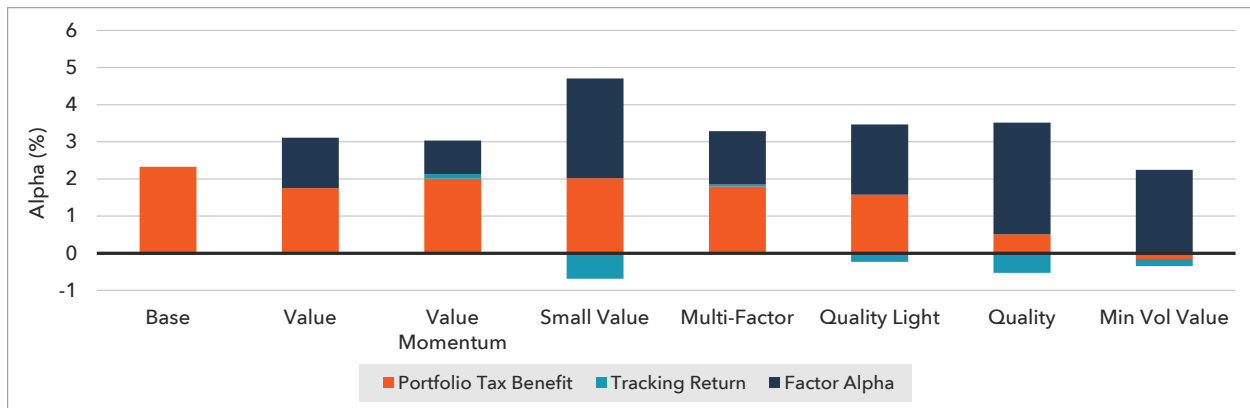
### A High-Level Look at Factor Tilts in the US and Global Markets

We begin with averages of simulated after-tax active returns of a tax-managed index and seven factor tilts in both the US and global markets.<sup>5</sup> In Figure 1, we show average factor alpha (orange bars). It is no surprise that factor alpha was positive, since investors' interest in these strategies derives, at least in part, from their historical success.

Tax alpha, the sum of tracking return (aqua bars) and portfolio tax benefit (dark blue bars) was positive, on average, in both US and global markets for the indexing (base) strategy, the beta 1 strategies—Value, Value Momentum, Small Value, and Multi-Factor—and the beta 0.9 strategy, Quality Light. For the low-risk strategies, Minimum Volatility plus Value and Quality, average tax alpha was negligible in the US. It was more substantial for the low-risk strategies in the global market.

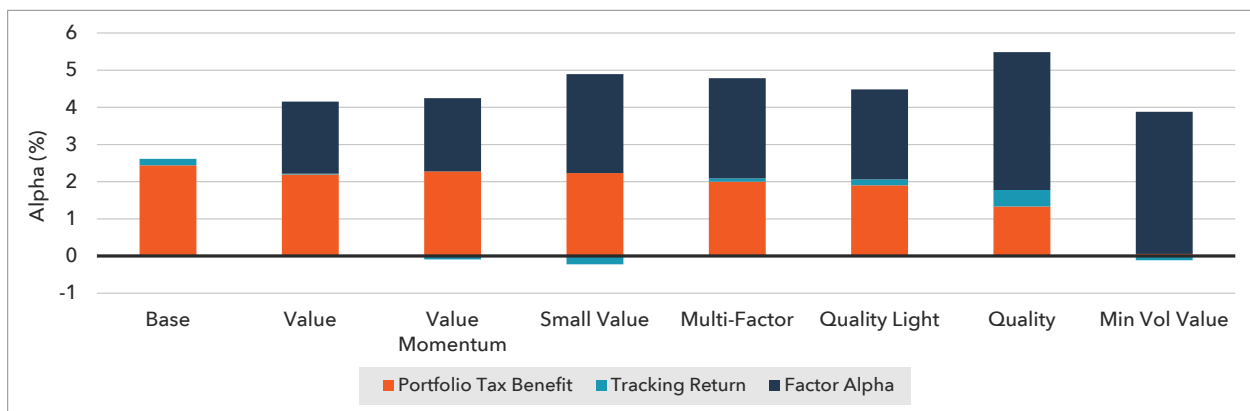
<sup>5</sup> The indexes used to measure performance vary by strategy. The details are in Table 1.

US Market



	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Index Return	5.39	5.39	5.79	5.79	5.70	5.39	5.39	5.39
After-Tax Active Return	2.28	3.08	3.03	4.02	3.28	3.24	2.99	1.90
Factor Alpha	0.00	1.36	0.91	2.68	1.43	1.89	3.01	2.24
Tax Alpha	2.28	1.72	2.13	1.34	1.85	1.34	-0.02	-0.35
Portfolio Tax Benefit	2.33	1.75	2.01	2.02	1.79	1.58	0.51	-0.16
Tracking Return	-0.05	-0.03	0.12	-0.69	0.06	-0.23	-0.53	-0.19

Global Market



	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Index Return	5.56	5.56	5.56	5.56	5.56	5.56	5.56	5.56
After-Tax Active Return	2.62	4.15	4.15	4.66	4.79	4.48	5.48	3.77
Factor Alpha	0.00	1.94	1.97	2.66	2.70	2.42	3.71	3.82
Tax Alpha	2.62	2.21	2.18	2.00	2.09	2.06	1.77	-0.06
Portfolio Tax Benefit	2.44	2.19	2.27	2.23	2.00	1.90	1.33	0.06
Tracking Return	0.18	0.02	-0.09	-0.23	0.09	0.16	0.44	-0.11

**Figure 1:** Ten-year average estate/donation after-tax active return for a tax-managed indexing strategy and seven factor tilts. Top panel: US market. Bottom panel: Global market. June 1995–December 2017. Source: Aperio Group, LLC.

For the rest of this paper, we focus on the US market. Companion charts for the global market are in Appendix A.

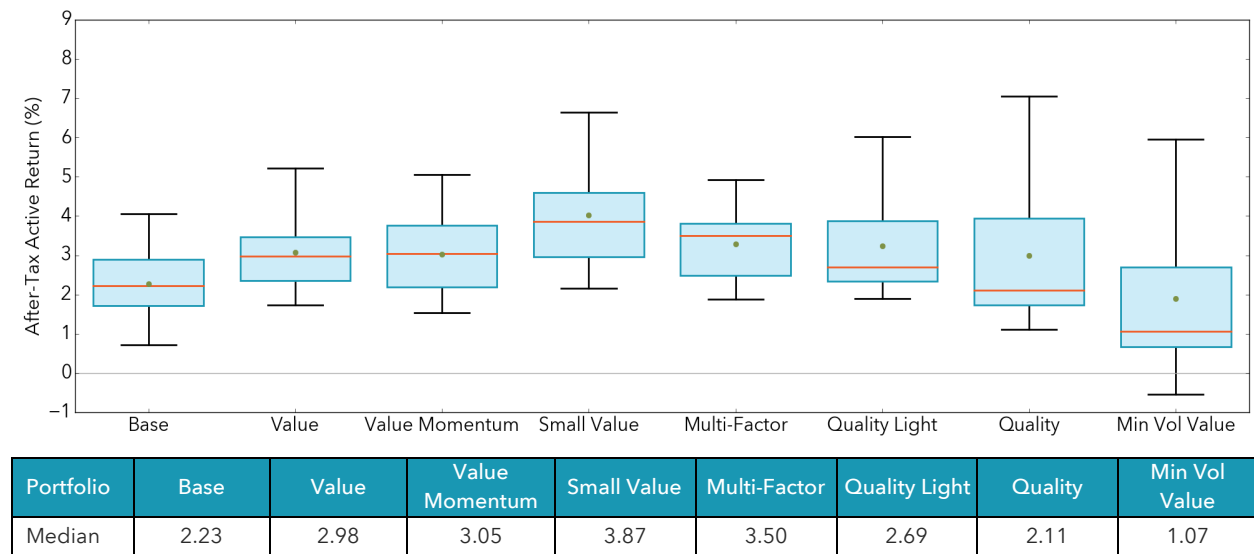
## A Deep Dive into the US Market

In the previous section, we looked at results averaged over 50 historical runs in both the US and global markets. In this section, we look in more detail at the US market. We consider ranges of after-tax active return over the 50 runs as well as breakdowns into factor and tax components. We look at the regime dependence of tax-managed factor tilts. Finally, we consider the incremental tracking error that tax-loss harvesting adds to the risk of a factor tilt relative to its benchmark.

### After-Tax Active Return

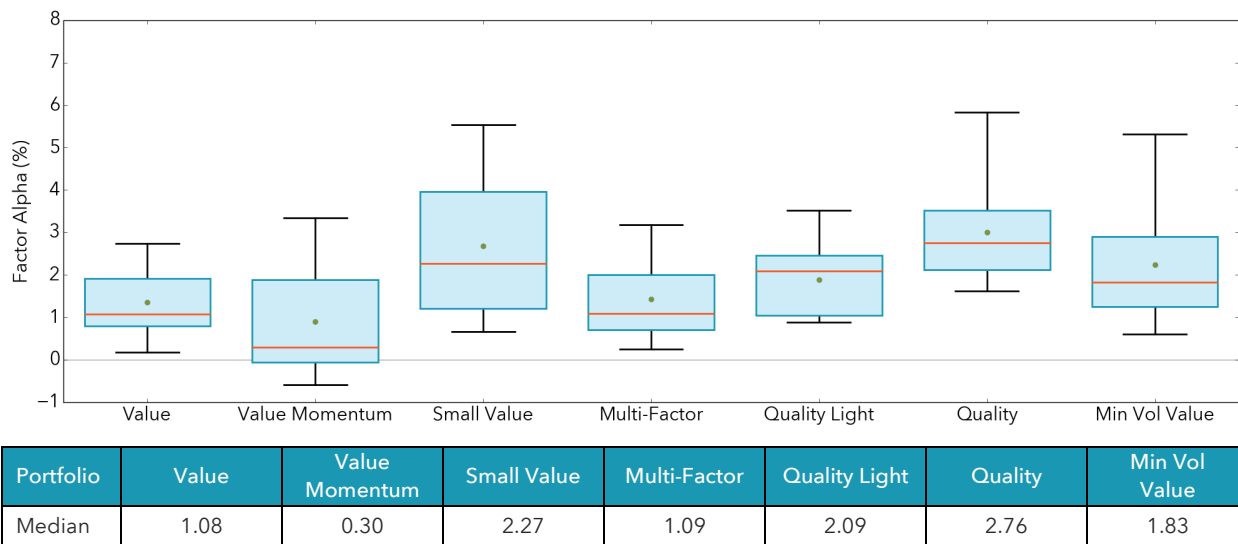
Figure 2 shows box plots of 10-year estate/donation after-tax active return for a tax-managed indexing strategy and seven factor tilts in the US market. Each box plot shows the full range of outcomes observed for 10-year strategies between June 1995 and December 2017. The orange line in each box is the median, and the dot marks the average. The top and bottom of the box correspond to the 75<sup>th</sup> and 25<sup>th</sup> percentile, respectively, and the maximum and minimum observations are marked by horizontal black lines.

Median values of after-tax active return ranged from 1.07% for Minimum Volatility plus Value to 3.87% for Small Value, and the widest ranges in performance came from the low-volatility strategies, Minimum Volatility plus Value and Quality. In subsequent charts, we decompose after-tax active return into its sources.



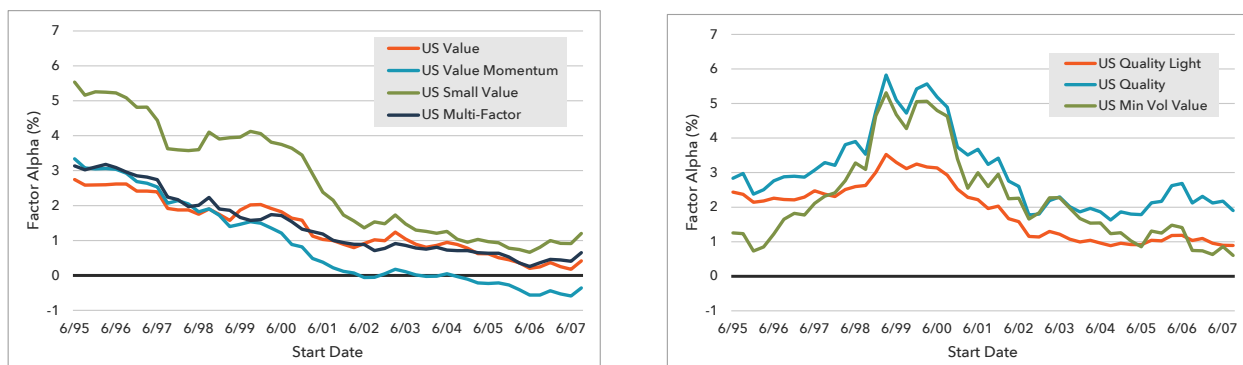
**Figure 2:** Ten-year estate/donation after-tax active return for a tax-managed indexing strategy and seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

Factor alpha, the component of after-tax active return due to factors, is shown for each of our strategies in Figure 3. The results vary, both in the medians and dispersion of outcomes. Factor alpha in the medium-volatility strategy, Quality Light, and the low-volatility strategies, Quality and Minimum Volatility plus Value, compared favorably to factor alpha in the beta 1 strategies. Of course, there is no guarantee that the low-volatility anomaly, or any anomaly for that matter, will persist in the future.



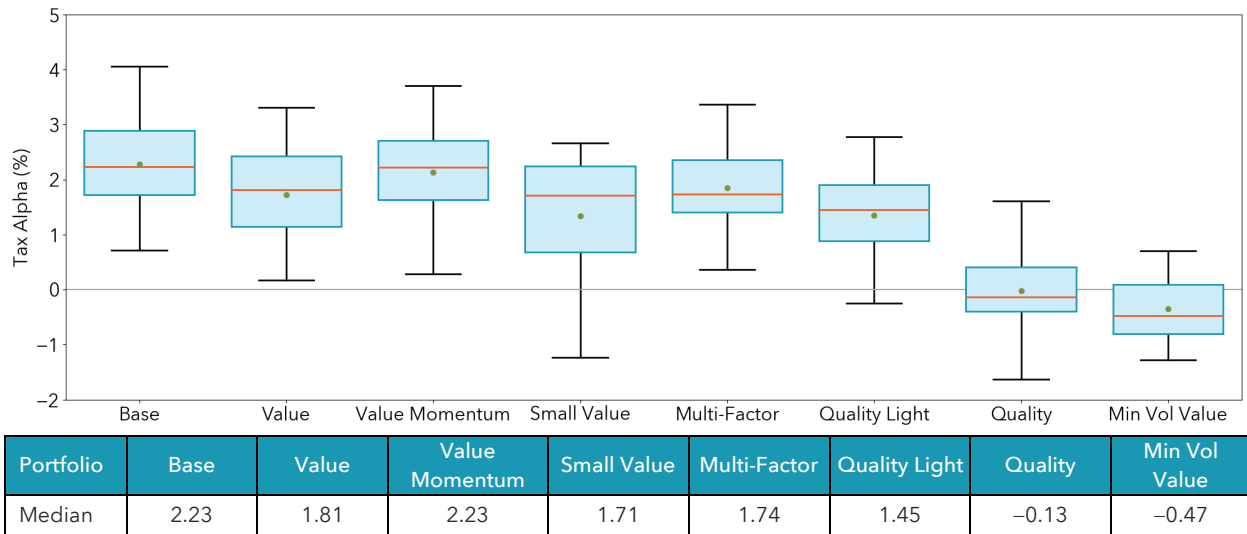
**Figure 3:** Ten-year factor alpha for seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

The box plots in Figure 3 show ranges of observed factor alpha at a 10-year horizon over our study period, but they neglect the temporal evolution of factor alpha. In Figure 4, we show rolling 10-year factor alpha for our seven tilts in the US market. It declined steadily for the beta 1 factor tilts since June 1995, and for the low-risk strategies since the turn of the millennium.



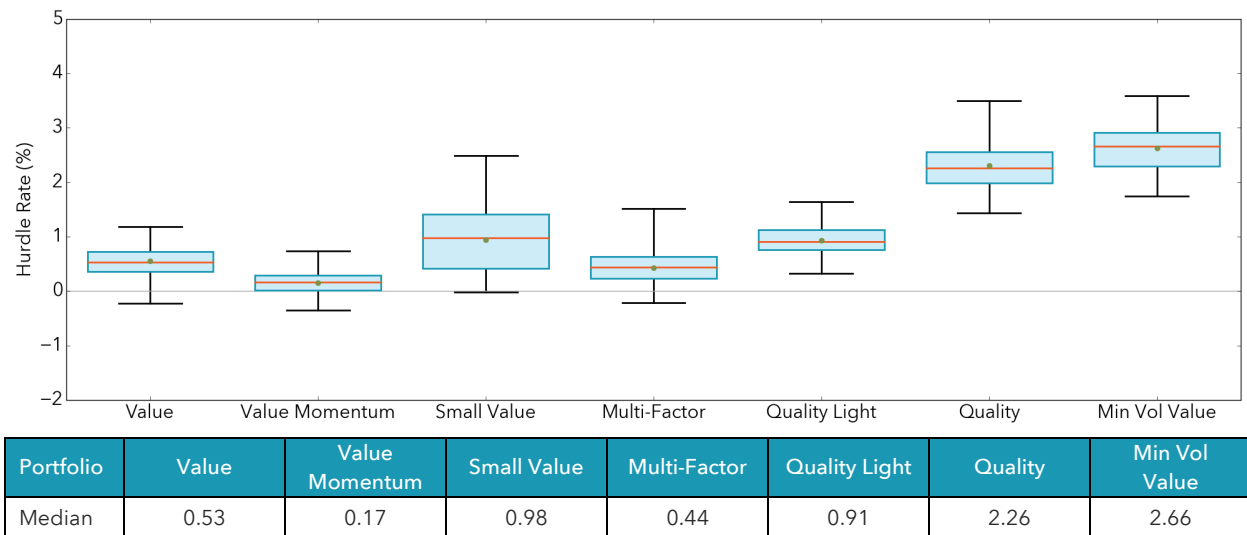
**Figure 4:** Rolling 10-year factor alpha for seven factor tilts in the US market. Source: Aperio Group, LLC.

We turn now to tax alpha. At a 10-year horizon, estate/donation tax alpha for the indexing strategy and seven factor tilts in the US market is shown in Figure 5. In contrast with the results for factor alpha, there was a pronounced difference between the lower-risk strategies and the beta 1 strategies, whose median tax alpha ranged from -0.47% for Minimum Volatility plus Value to 2.23% for Value Momentum. This difference is consistent with the notion that tax alpha tends to be less abundant in portfolios that are smaller or composed of less-volatile stocks.



**Figure 5:** Ten-year estate/donation tax alpha for a tax-managed indexing strategy and seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

An investor can combine information about differences in tax alpha with his level of conviction in factors to select a strategy. The salient metric is the *hurdle rate*, the difference between tax alpha in an indexing strategy and a factor strategy. In order for a tilt to be a rational choice for a taxable investor, its factor alpha must exceed its hurdle rate.

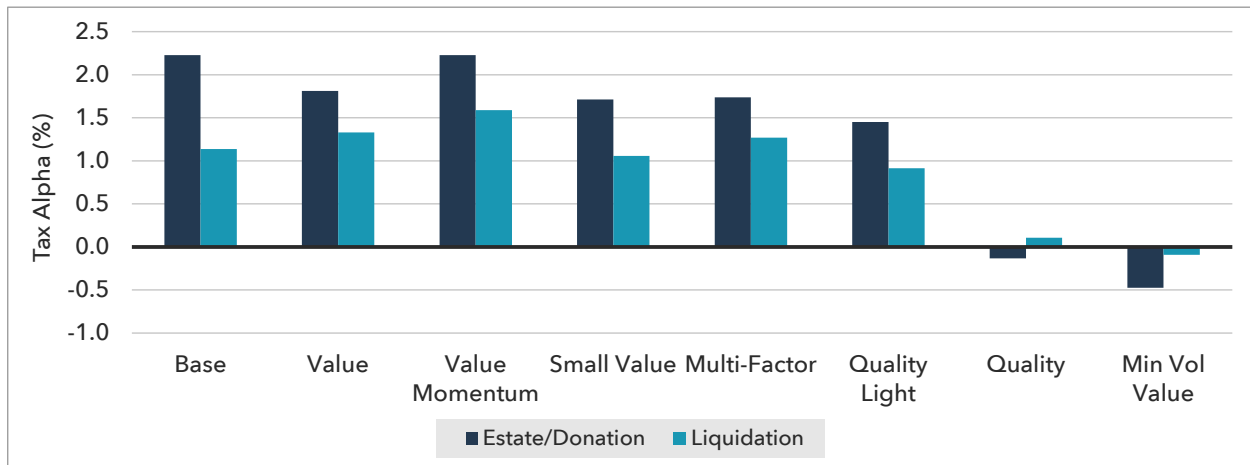


**Figure 6:** Ten-year estate/donation hurdle rates for seven tax-managed factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.



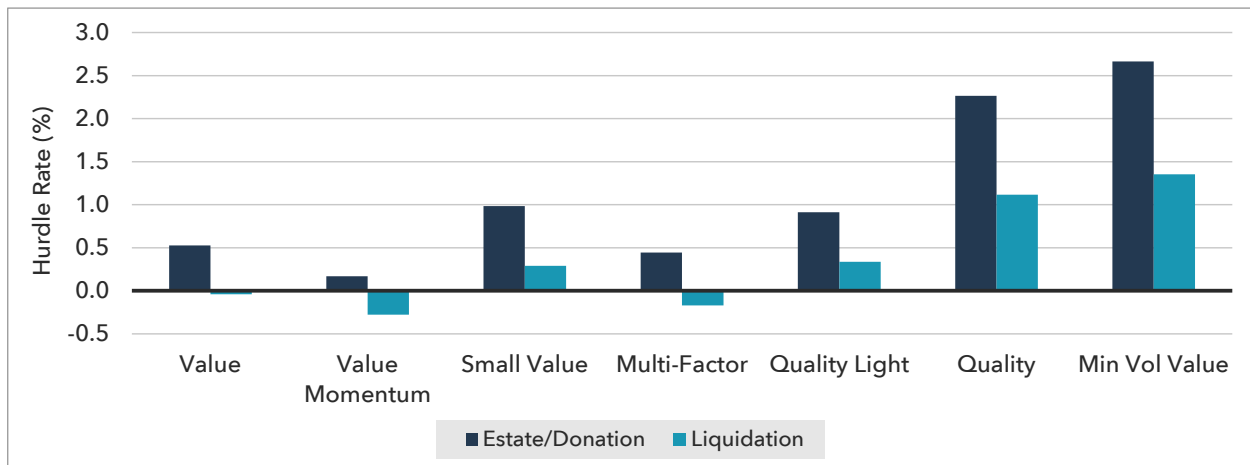
Figure 6 shows ranges of hurdle rates faced by seven factor tilts over the period June 1995 to December 2017. Hurdle rates tended to be higher in strategies with lower risk, which is a simple restatement of the results in Figure 5, which show lower tax alpha for strategies with lower risk. Strategies with the lowest hurdle rates included Value Momentum, which is compatible with loss harvesting.

The results shown in Figures 1 through 6 are for the estate/donation disposition, in which taxes are never paid. In the liquidation disposition, where taxes are merely deferred, tax alpha tended to be lower, as illustrated in Figure 7. Exceptions do arise, however: liquidation tax alpha can be higher than estate/donation tax alpha when the benchmark tax payments are unusually high. Figure 8 shows median hurdle rates for factor tilts in the two dispositions. Hurdle rates were lower for the liquidation disposition in all the strategies we considered.



Portfolio	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Estate/Donation	2.23	1.81	2.23	1.71	1.74	1.45	-0.13	-0.47
Liquidation	1.14	1.33	1.59	1.06	1.27	0.91	0.11	-0.09

**Figure 7:** Median tax alpha for a 10-year tax-managed indexing strategy and seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

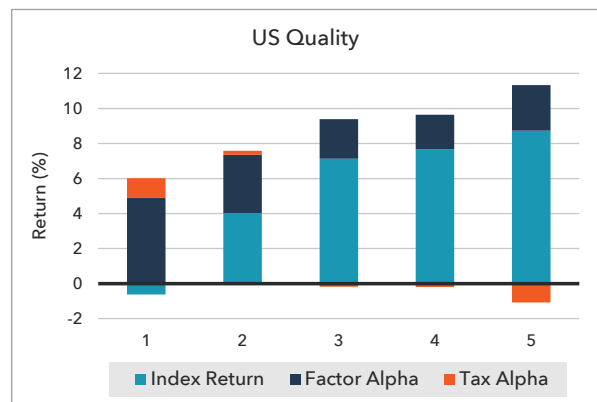
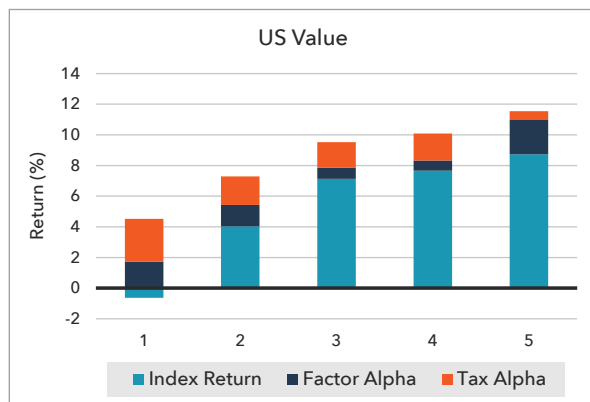


Portfolio	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Estate/Donation	0.53	0.17	0.98	0.44	0.91	2.26	2.66
Liquidation	-0.04	-0.28	0.29	-0.17	0.34	1.12	1.35

**Figure 8:** Median hurdle rates for seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

### Regimes

Like put options and other defensive strategies, loss harvesting has been most effective in turbulent, declining markets.<sup>6</sup> We quantify this for two of our strategies, Value and Quality, which represent a beta 1 and a low-risk strategy. We sorted 10-year returns to Value and Quality into quintiles on the basis of performance of the underlying index. In Figure 9, we show the average index return along with average tax alpha and factor alpha for the tilts. For both strategies, tax alpha (orange bars) tended to improve as index performance diminished. Factor alpha in Quality showed a similar tendency, consistent with the defensive nature of low-risk strategies. For Value, in contrast, there was no apparent relationship between factor alpha and index return.



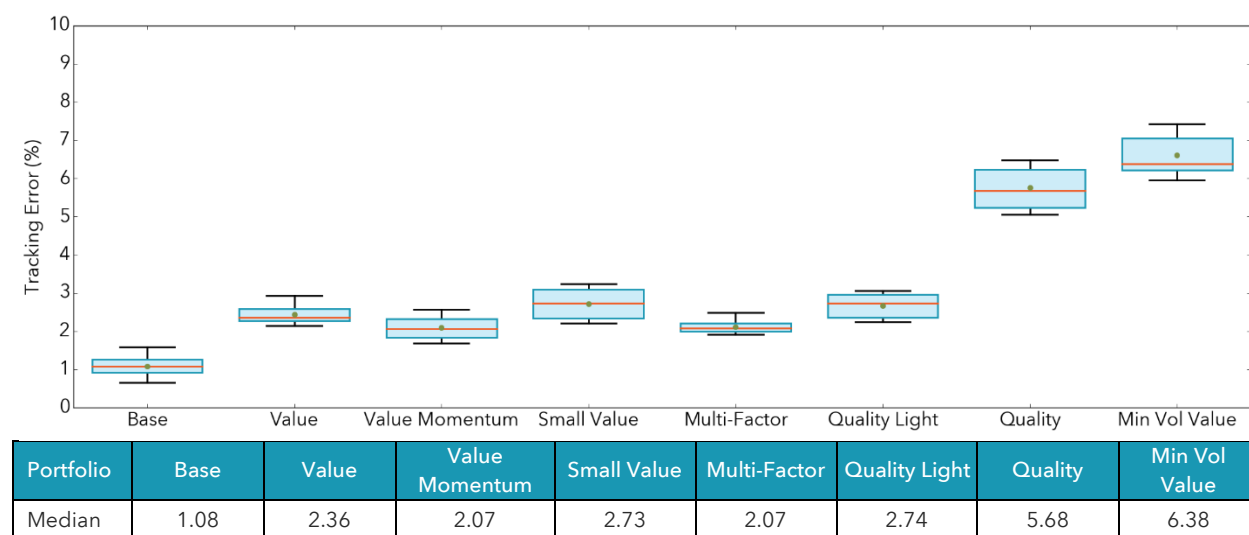
<sup>6</sup> The association of volatility and market decline is known as the leverage effect, and it was first documented in Black (1976).

**Figure 9:** Ten-year index return, factor alpha, and tax alpha of tax-managed factor tilts in the estate/donation disposition averaged over quintiles by index return, June 1995–December 2017. Left panel: US Value. Right panel: US Quality. Source: Aperio Group, LLC.

**Risk**

Loss harvesting tends to increase tracking error, and there are several ways to measure the impact. We look at incremental tracking error, the increase in the width of the distribution of returns relative to a diversified benchmark, that loss harvesting adds to the risk of a factor tilt.<sup>7</sup> A baseline is in Figure 10, which shows realized tracking error of tax-managed tilts relative to their benchmarks.<sup>8</sup> A strategy with higher tracking error has a greater chance of substantial underperformance.

Tracking error was higher for the low-risk strategies, illustrating the unavoidable trade-off between absolute and relative risk in equity markets. The lowest tracking error was achieved for the tax-managed index, which was not afflicted by tilt constraints.

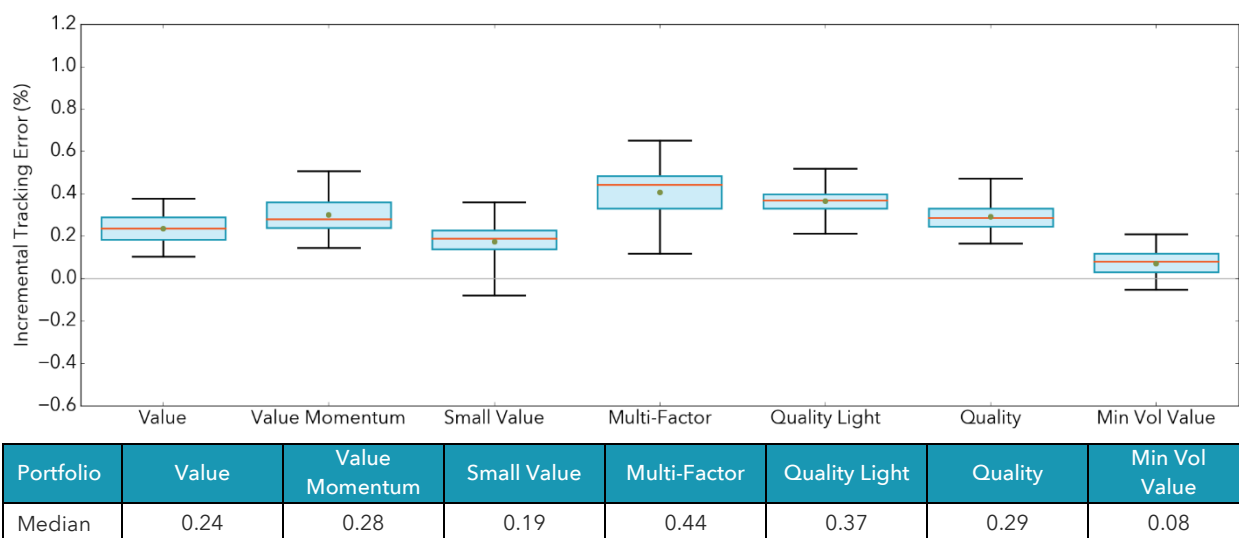


**Figure 10:** Realized tracking error for a tax-managed indexing strategy and seven factor tilts over a 10-year horizon in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

In Figure 11, we look at the difference in tracking error between tax-managed and tax-indifferent factor tilts. The difference is the incremental risk due to loss harvesting. Note the differences in scale between Figure 10, measured in percent, and in Figure 11, measured in tenths of a percent. The median incremental tracking error was 0.44% or less for all US market strategies. A taxable investor in a factor tilt must increase her risk budget accordingly.

<sup>7</sup> Tracking error is not additive, so there can be distortions associated with incremental tracking error.

<sup>8</sup> Tracking error is the standard deviation of the difference in return between a portfolio and its benchmark before taking account of taxes. Incorporation of tax alpha introduces a substantial right skew, which calls for a nuanced risk assessment.



**Figure 11:** Difference in realized tracking error between tax-managed and tax-indifferent factor tilts at a 10-year horizon in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

### The Relative Abundance of Tax Alpha in Beta 1 Strategies

Figures 5 and 6 show that tax alpha was more abundant in strategies with higher absolute risk. Here, we provide two candidate explanations for this finding.

Loss harvesting tends to be less effective when the universe of securities is sufficiently small, or when the securities themselves are sufficiently tame. This is supported by Table 3, which shows the median number of stocks in each US strategy over the study period, as well as the median strategy volatility.

	Number of Securities	Forecast Volatility (%)
Base	292	15.13
Value	197	14.80
Value Momentum	314	15.28
Small Value	441	15.30
Multi-Factor	265	14.98
Quality Light	236	13.28
Quality	85	11.43
Min Vol Value	82	10.64

**Table 3:** Median number of stocks and median forecast volatility for a tax-managed indexing strategy and seven factor tilts in the US market. June 1995–December 2017. Source: Aperio Group, LLC.

## Summary

Factor tilts are attractive to taxable investors, but the turnover required to maintain factor tilts tends to reduce tax alpha. We examined this trade-off in seven factor tilts at a 10-year horizon over the period June 1995 to December 2017.

Tax alpha in beta 1 factor tilts—Value, Value Momentum, Small Value, and Multi-Factor—was on par with tax alpha in an indexing strategy. In the US market, median values of tax alpha for these strategies ranged from 1.71% to 2.23% for the estate/donation disposition and from 1.06% to 1.59% for the liquidation disposition. Tax alpha was diminished in low-volatility strategies such as Minimum Volatility plus Value and Quality.

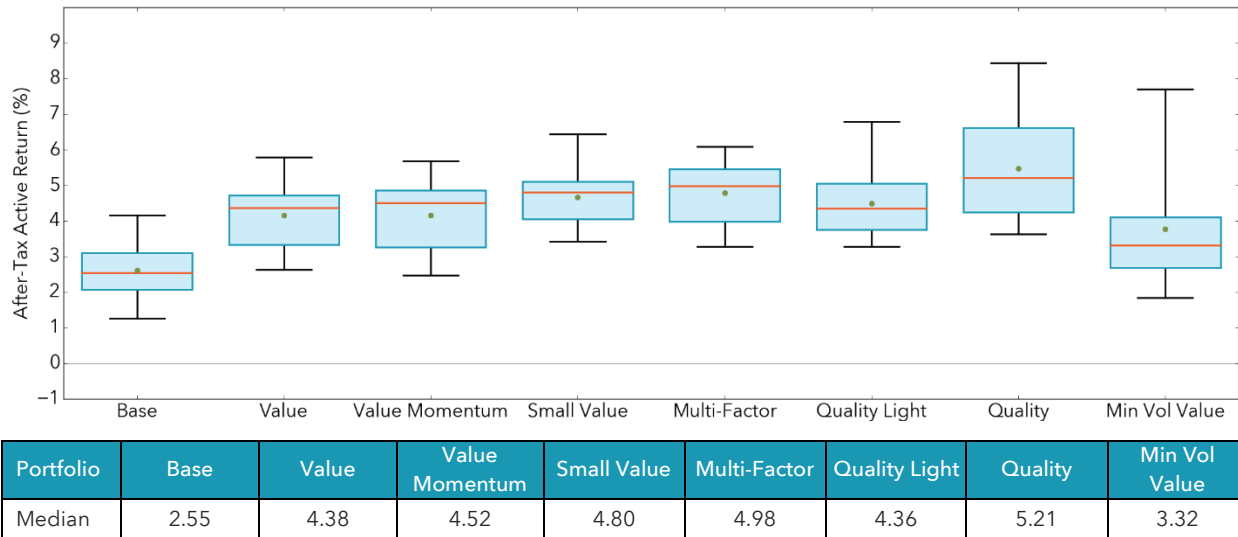
The difference between tax alpha in an indexing strategy and tax alpha in a factor tilt is the hurdle rate: the factor alpha required to make the strategy a rational choice for a taxable investor. In the US market, median hurdle rates were highest for Minimum Volatility plus Value, at 2.66% in the estate/donation disposition and 1.35% in the liquidation disposition. More generally, they were higher in low-risk strategies than in beta 1 strategies, for which the median values were in the neighborhood of 0%.

At the same time, the median incremental tracking of error due to loss harvesting was 0.44% or less for all US market strategies.

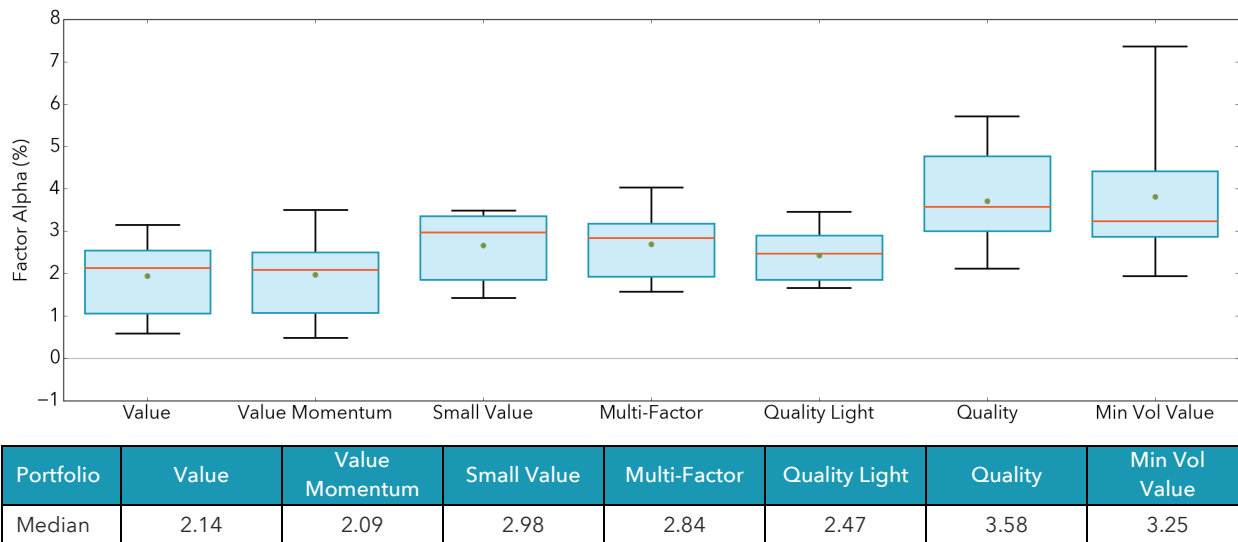
The decision about whether to invest in a tax-managed factor tilt and which tilt to choose depends on the investor. At one extreme, an investor with low tolerance for volatility but a willingness to deviate from a benchmark and make do with a little tax alpha might choose Minimum Volatility plus Value. At the other extreme, an investor who is averse to tracking error, tolerant of volatility, and very interested in tax alpha might choose a beta 1 factor tilt. The choice of a factor strategy over indexing requires conviction in the performance of the factor in question, and as usual, one size does not fit all.

### Appendix A: Companion Figures for Global Markets

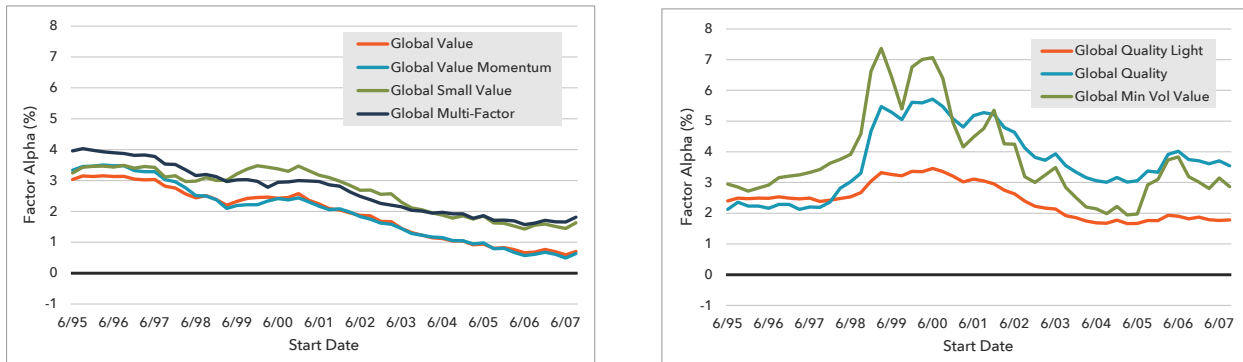
The charts in this Appendix pertain to tax-managed factor tilts in the global market. The study period and methodology are the same as for the US market, and the main findings in the two markets are similar. The most pronounced difference is that tax alpha tended to be higher in global markets, especially for low-risk strategies. This was mainly due to higher tracking return, which is primarily attributable to chance. The higher tax alpha was offset, to some degree, by lower index return.



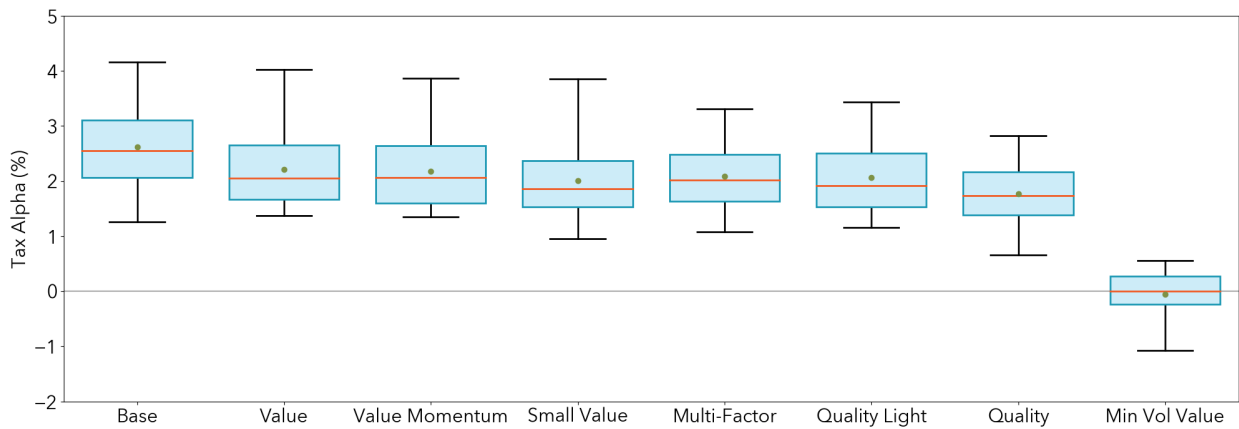
**Figure 2G:** Ten-year estate/donation after-tax active return for a tax-managed indexing strategy and seven factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 2.



**Figure 3G:** Ten-year estate/donation factor alpha for seven factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 3.

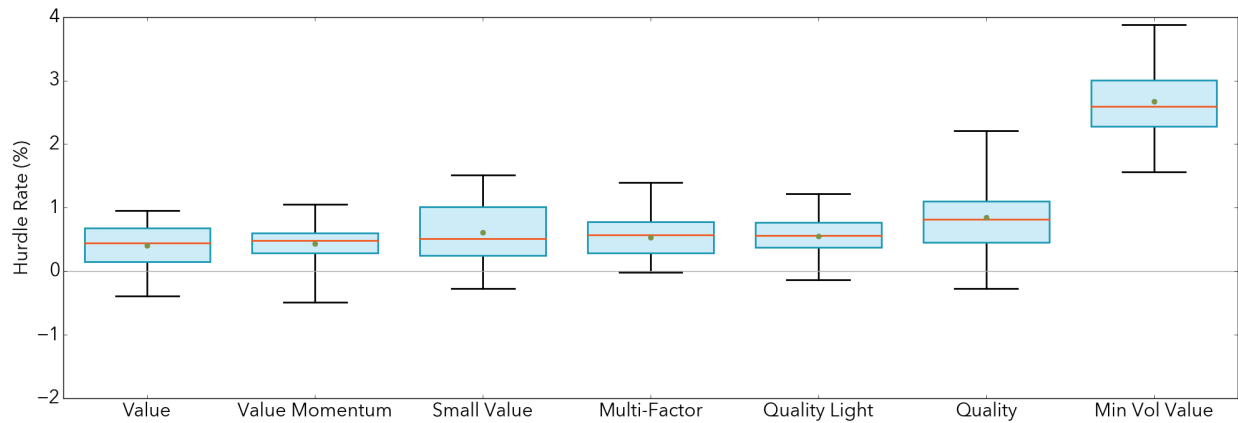


**Figure 4G:** Rolling 10-year estate/donation factor alpha for seven factor tilts in the global market. Source: Aperio Group, LLC. This figure is analogous to Figure 4.



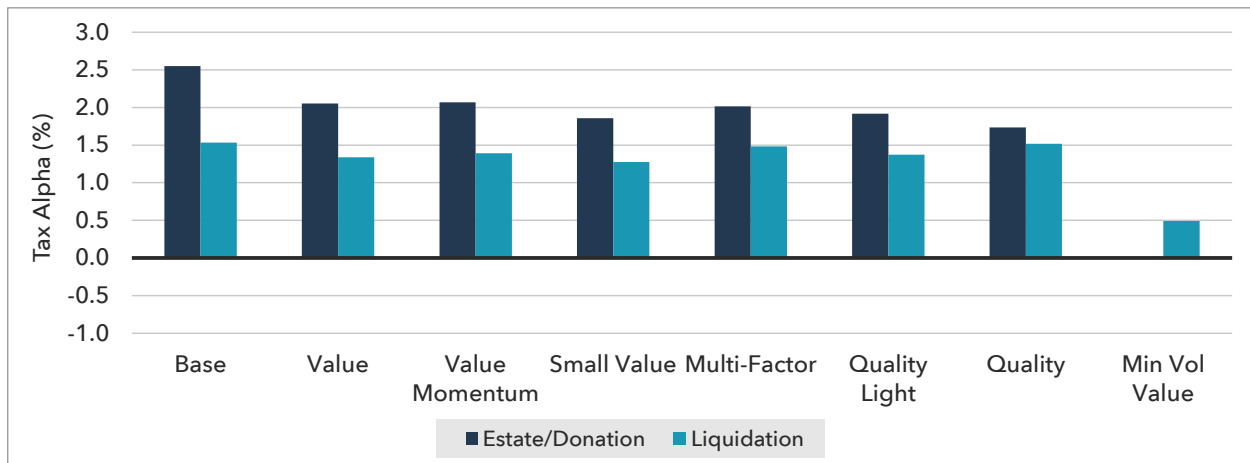
Portfolio	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Median	2.55	2.05	2.07	1.86	2.01	1.92	1.73	0.01

**Figure 5G:** Ten-year estate/donation tax alpha for a tax-managed indexing strategy and seven factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 5.



Portfolio	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Median	0.44	0.48	0.51	0.57	0.56	0.82	2.59

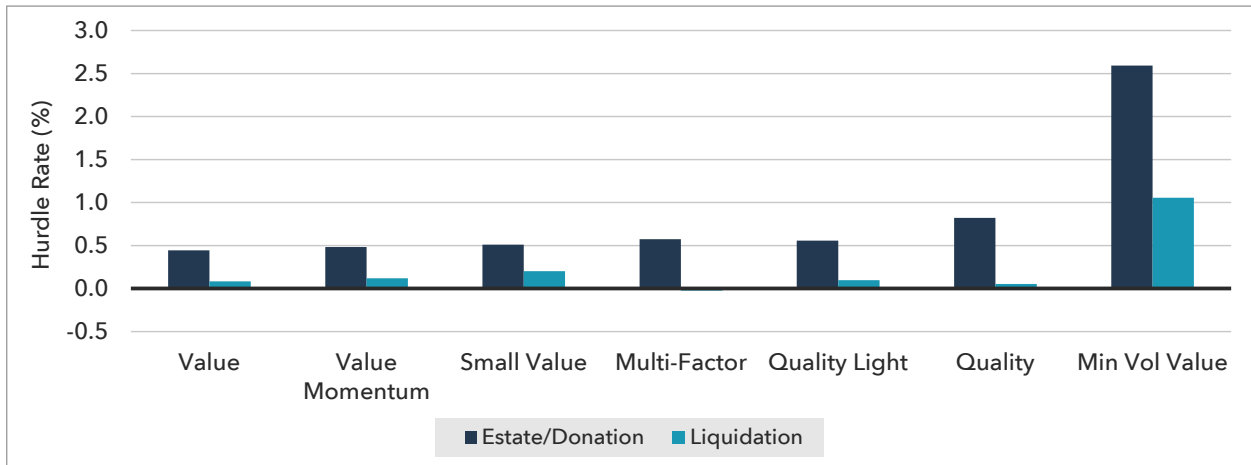
**Figure 6G:** Ten-year estate/donation hurdle rates for seven tax-managed factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 6.



Portfolio	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Estate/Donation	2.55	2.05	2.07	1.86	2.01	1.92	1.73	0.01
Liquidation	1.53	1.34	1.39	1.28	1.48	1.37	1.52	0.49

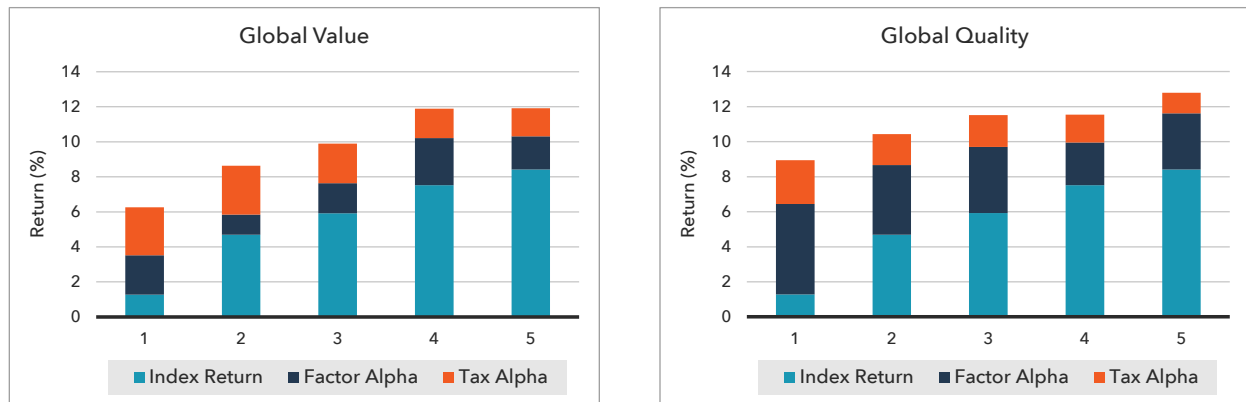
**Figure 7G:** Median tax alpha for a 10-year tax-managed indexing strategy and seven factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 7.



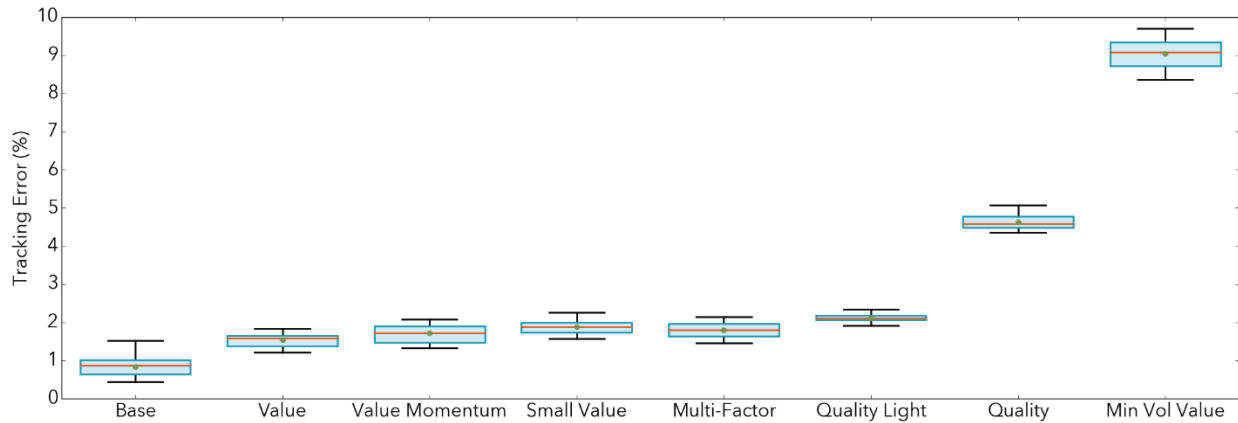


Portfolio	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Estate/Donation	0.44	0.48	0.51	0.57	0.56	0.82	2.59
Liquidation	0.08	0.12	0.20	-0.03	0.10	0.05	1.05

**Figure 8G:** Median hurdle rates for seven factor tilts in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 8.

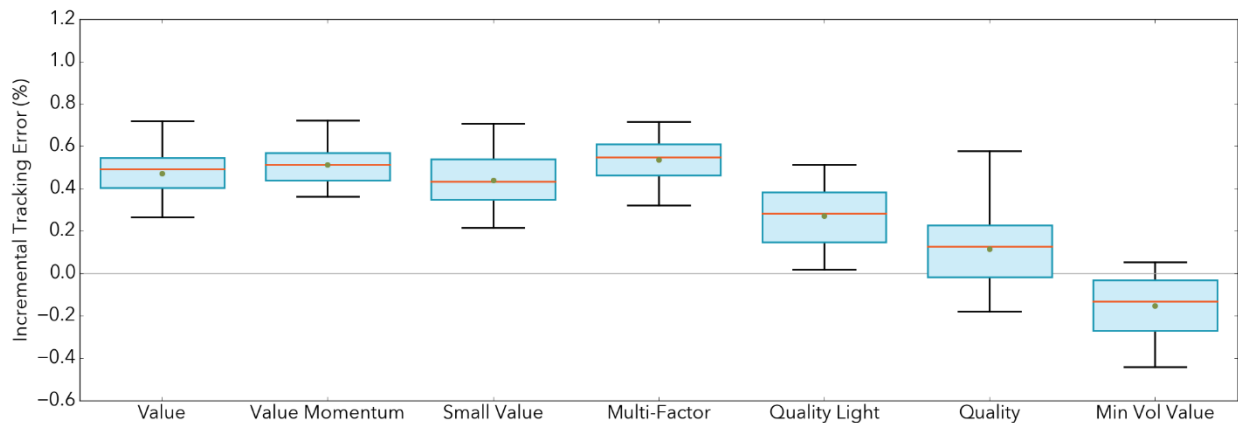


**Figure 9G:** Ten-year index return, factor alpha, and tax alpha of tax-managed factor tilts in the estate/donation disposition averaged over quintiles by index return, June 1995–December 2017. Left panel: Global Value. Right panel: Global Quality. Source: Aperio Group, LLC. This figure is analogous to Figure 9.



Portfolio	Base	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Median	0.87	1.59	1.72	1.88	1.80	2.11	4.58	9.09

**Figure 10G:** Realized tracking error for a tax-managed indexing strategy and seven factor tilts over a 10-year horizon in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 10.



Portfolio	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Median	0.49	0.51	0.43	0.55	0.28	0.13	-0.13

**Figure 11G:** Difference in realized tracking error between tax-managed and tax-indifferent factor tilts at a 10-year horizon in the global market. June 1995–December 2017. Source: Aperio Group, LLC. This figure is analogous to Figure 11.

	Number of Securities	Forecast Volatility (%)
Base	1,194	14.74
Value	560	14.78
Value Momentum	487	14.79
Small Value	598	14.77
Multi-Factor	387	14.75
Quality Light	559	13.29
Quality	212	11.20
Min Vol Value	116	9.01

**Table 3G:** Median number of stocks and median forecast volatility for a tax-managed indexing strategy and seven factor tilts in the Global market. June 1995–December 2017. Source: Aperio Group, LLC. This table is analogous to Table 3.

## Appendix B: Strategy Construction

Aperio’s ATBAT (After-Tax Back-Testing Analysis Tool) platform constructs portfolios with Barra models and software. We used Barra US Total Market Equity Model for Long-Term Investors (USSLOWL) for all US back-tests, and Barra Global Total Market Equity Model for Long-Term Investors (GEMTL) for all global back-tests. The optimizer settings that we used in this study are included below. This appendix does not include comprehensive documentation of Barra. More information is available in Barra documentation.<sup>9</sup>

### General Settings

Model	
Barra Model	USSLOWL for US strategies; GEMTL for global strategies
Optimization	
Type	After-Tax
Initial Cash	10,000,000
Transactions	Buy From Universe Only
Transaction Costs	
Buy Costs (%)	0.06%
Sell Costs (%)	0.06%
Holdings	
General Holding Upper Bound (%)	100 for Base; 1.5 for Min Vol Value; 3 for other tilts
General Holding Lower Bound (%)	0
Taxes	
Short Term Tax Rate (%)	40.8
Long Term Tax Rate (%)	23.8
Tax Lot Relief	Min Tax Liability

<sup>9</sup> Documentation on Barra models is in Bayraktar et al. (2014) and Morozov et al. (2015).

## Reference Portfolios

### US

	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Objective	Min TE	Min TE	Min TE	Min TE	Min TE	Min TE	Min Var
Benchmark	S&P 500	R3000	R3000	R1000	S&P 500	S&P 500	Cash
Universe	R1000	R3000	R3000	R1000	R1000	R1000	R3000
Target Beta	1	1	1	1	0.9	0.75	N/A
Sector Constraints	benchmark +/- 1	benchmark +/- 1	benchmark +/- 1	benchmark +/- 2	benchmark +/- 10	benchmark +/- 10	benchmark +/- 5
Industries				benchmark +/- 5			
<b>Risk Index Tilts</b>							
Book-to-Price	+	+	+	+			+
Earnings Yield	+	+	+	+	+	+	+
Momentum		+		+			
Residual Volatility					+	+	
Beta					+	+	
Earnings Quality					+	+	
Profitability				+	+	+	
Management Quality				+			
Leverage					+	+	
Size			+	+	+	+	

### Global

	Value	Value Momentum	Small Value	Multi-Factor	Quality Light	Quality	Min Vol Value
Objective	Min TE	Min TE	Min TE	Min TE	Min TE	Min TE	Min Var
Benchmark	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	Cash
Universe	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI	MSCI ACWI
Target Beta	1	1	1	1	0.9	0.75	N/A
Sector Constraints	benchmark +/- 1	benchmark +/- 1	benchmark +/- 1	benchmark +/- 2	benchmark +/- 10	benchmark +/- 10	benchmark +/- 5
Industries				benchmark +/- 5			
Country Constraints							benchmark +/- 5
<b>Risk Indexes</b>							
Book-to-Price	+	+	+	+	+	+	+
Earnings Yield	+	+	+	+	+	+	+
Momentum		+		+			
Residual Volatility					+	+	
Beta					+	+	
Earnings Variability					+	+	
Earnings Quality					+	+	
Profitability				+	+	+	
Investment Quality				+			
Leverage					+	+	
Size			+	+	+	+	

## References

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## Disclosure

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With respect to the description of any investment strategies, simulations, or investment recommendations, we cannot provide any assurances that they will perform as expected and as described in our materials. Past performance is not indicative of future results. Every investment program has the potential for loss as well as gain.

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The S&P 500® Index is an equity benchmark for US stock performance. It is a capitalization-weighted index covering 500 large US companies chosen by Standard & Poor's for market size, liquidity, and industry group representation.

The Russell 1000® Index is an equity benchmark for US stock performance. It is a capitalization-weighted index covering the largest 1,000 publicly-traded US stocks. The index represents approximately 90% of the total market capitalization of the US stock market.

The Russell 3000® Index is an equity benchmark for US stock performance. It is a capitalization-weighted index covering the largest 3,000 publicly-traded US stocks. The index represents approximately 98% of the total market capitalization of the US stock market.

The MSCI ACWI Index is an equity benchmark for global stock performance. It is a capitalization-weighted index covering large and mid-sized companies. The index includes approximately 2,500 stocks from 23 developed-market countries and 24 emerging-market countries.