
RESTORING VALUE TO MINIMUM VARIANCE

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A long-only investable minimum variance strategy outperformed the S&P 500 over the four decades from January 1973 to December 2012. Through the lens of a factor model, we show that this outperformance can be largely attributed to implicit style bets. Specifically, minimum variance has thrived by tilting toward stocks that have lower market capitalization and volatility, and a higher ratio of earnings to price. As funds have poured into minimum variance in the wake of the financial crisis, and plausibly as a consequence of this trend, the value tilt has disappeared and a momentum tilt has emerged. This suggests that the cost of entry to minimum variance is at a historic high. We show how the value tilt can be restored to minimum variance by targeting specific exposures, and that there was a substantial long-term benefit to the restoration at most recent points of entry to the strategy.



Over the past few years, investors have been drawn to strategies that promise to lower equity portfolio risk. Their motivations have included (1) a heightened risk sensitivity after the painful experience of the 2008–2009 market meltdown, (2) downside protection as market valuations climbed and U.S. indexes set new records, (3) an urgent need to find a substitute for traditional lower risk asset classes like bonds and cash since

those now offer such unattractive real returns and (4) empirical evidence of higher returns contrary to traditional portfolio theory. However, while the historical data may make a compelling case for higher returns with lower risk, investors need to go deeper into the explanations behind the historical performance.

This article examines one popular low-risk strategy, minimum variance, which optimizes a basket of stocks to deliver the lowest possible portfolio variance. In back tests, this strategy earned a higher return than the market during the period from 1973 to 2012. We explore the factor exposures behind the excess return and how those exposures have changed over time. In particular,

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the strategy's exposure to the value factor (as defined by higher earnings yield) has shifted significantly from what had been a consistent exposure to lower valuation stocks to what is now the opposite, exposure to stocks with valuations higher than the market's. While the exact cause of the recent shift from cheap to expensive remains unclear, it is consistent with an increase in popularity of low-risk strategies. Some of the biggest ETF launches of the past three years included low-risk funds from iShares (USMV) and Powershares (SPLV).¹ The popularity of low-risk strategies may be at least partially responsible for the increased valuations for low-volatility stocks. In effect, investors may be starting to pay premium prices for lower risk stocks that used to be relatively inexpensive.

However, investors who seek reduced risk may not need to bear expensive valuations. By adjusting the conventional minimum variance strategy to control factor exposures it is possible to afford investors the benefit of low-volatility while mitigating the risk of overpaying for lower volatility stocks.

1 A brief overview of low-risk investing

Markowitz (1952) observed that a minimum variance portfolio is optimal if all securities have the same expected returns. Empirical analysis of low-risk strategies dates back at least as far as Black *et al.* (1972), which found that in a frictionless market, low-beta stocks tend to have higher risk-adjusted returns than do high-beta stocks.² That study questioned the validity of the Capital Asset Pricing Model (CAPM), which was a burning research topic at the time, and it suggested that the addition of a second factor that is long low-beta stocks and short high-beta stocks could lead to a better pricing model.

Frazzini and Pedersen (2013) documented the low-beta anomaly in different markets and

over different time periods. Similar anomalies have been observed for total volatility, leading to minimum variance, and also for idiosyncratic volatility. Clarke *et al.* (2011) explain mathematically that there is substantial overlap among different types of low-risk investments, so the persistence of the anomaly across different risk measurements is not a coincidence.

Though the empirical results have shown consistency, there remains disagreement about the causes of low-risk anomalies. In a simple economy consisting of three coconut farms, Markowitz (2005) showed that if some investors are leverage constrained and others are not, the market portfolio will be inefficient and low-volatility portfolios will be more diversified than high-volatility portfolios. Frazzini and Pedersen (2013) argued that leverage-constrained investors are relegated to high-beta securities, so they bid up their prices. Investors with greater latitude can take advantage of levered low-beta securities, for which there is less demand. Cowan and Wilderman (2011) pointed out that in an abrupt market downturn, a levered low-beta portfolio can become bankrupt whereas the loss to the high-beta portfolio will be attenuated. In other words, investors in a high-beta portfolio are buying insurance, and they pay for it by sacrificing expected return.

Using the analysis of the covariance between beta and market return in Lewellen and Nagel (2006), Sefton *et al.* (2011) explained the low-risk anomaly in terms of the asymmetric response of the market to good and bad news. In volatile markets, betas are dispersed and low-beta stocks heavily outperform high-beta stocks. However, in calm markets, betas are concentrated and low-beta stocks underperform, but only by a little. Sefton *et al.* (2011) argued that in the long run, low-beta stocks dominate.

A behavioral explanation of the success of low-risk investing is in Baker *et al.* (2011), who drew an analogy between penny stocks, which tend to be quite volatile, and lottery tickets. Both are overpriced by rational standards. However, a chance, no matter how small, of an enormous future payoff can loom larger than the fact that the expected future payoff is not worth its price. This optimism is compounded by selective memory. We easily recall the big, successful enterprises like Apple that were once tiny. It is more difficult to remember the countless tiny enterprises that failed.

Crill and Davis (2012) and Scherer (2011) pointed out value and size tilts in minimum variance strategies and suggest that these strategies may be nothing more than an inefficient means to capture factor risk premiums. In the discussion below, we explore the ways in which risk factors contribute to the performance of minimum variance.

2 Minimum variance from a factor perspective

Factor models are an important outgrowth of the CAPM, and they are powerful tools for understanding portfolio performance, as well as for forecasting expected return and risk. In the following analysis we use the Barra USE3 factor model to analyze the difference between minimum variance and the S&P 500, which is the benchmark for our analysis. The USE3 model incorporates both industry and style factors.

Style factors, such as size, volatility, value, and momentum, play an important role in the analysis of minimum variance. Size measures market capitalization, and there is a documented, if controversial, propensity for smaller stocks to have higher returns than larger stocks. Volatility measures the tilt toward riskier stocks. Value is based on the ratio of earnings to price and it can be used to identify bargains in the marketplace.

Momentum is the tendency for trends to continue. Standard references on these factors are Fama and French (1992) and Carhart (1997).

We examine minimum variance over the four decades from January 1973 to December 2012. The S&P 500 is the optimization universe for minimizing variance, and this choice distinguishes our study from those based on a larger universe and may involve less liquid stocks. Our minimum variance strategy is long only³ and rebalanced quarterly, and it is optimized to minimize portfolio standard deviation while adhering to certain constraints.⁴ This strategy is roughly comparable to one investors can easily implement through an ETF from iShares, the MSCI U.S. Minimum Volatility fund that had high asset inflows in early 2013.

2.1 Minimum variance outperformed the S&P 500 benchmark

Figure 1 shows cumulative return to minimum variance and the S&P 500 between January 1973 and December 2012. The substantial outperformance of minimum variance is striking, especially in view of its low turnover. The summary

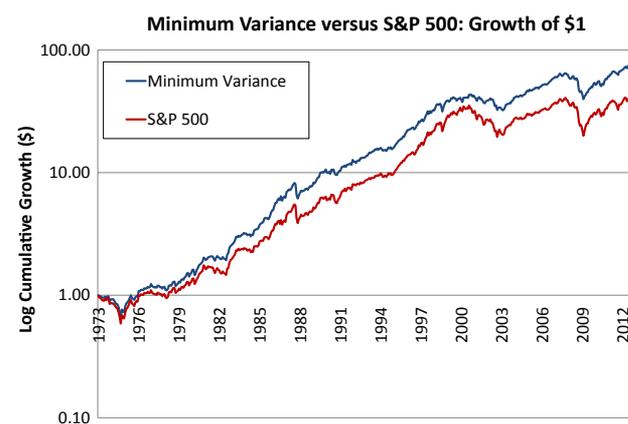


Figure 1 Log cumulative return to minimum variance and S&P 500. The outperformance of minimum variance has been pronounced since the turn of the century.

Table 1 Minimum variance versus S&P 500, 1973–2012.

Strategy	Minimum variance	S&P 500
Annualized returns	11.44%	9.80%
Return difference vs. S&P 500	+1.64%	0.00%
Annual standard deviation	12.89%	15.68%
Beta	0.78	1.00
Tracking error	5.17%	0.00%
Max. down year	–26.46%	–36.97%
Upside capture	86.7%	100.0%
Downside capture	70.3%	100.0%
Sharpe ratio	0.5	0.34

statistics in Table 1 show that since 1973, minimum variance had higher annualized return than the S&P 500, 11.4% versus 9.8%, and lower annualized volatility, 12.9% versus 15.7%. On a risk-adjusted basis, minimum variance had a Sharpe ratio of 0.50, whereas the S&P 500 had a Sharpe ratio of 0.34. Thus our back-tested results confirm research published on the empirical anomaly of earning a higher return while bearing lower risk.

2.2 *Style factors drove historical outperformance*

Table 2 shows that most of the active return of minimum variance relative to the S&P 500 benchmark is due to style factors, which contributed an average of 1.20% per year. As shown in Table 3, size proved the biggest and the most consistent contributor to active return over the study period, with the size tilt negative across time, i.e., the minimum variance portfolio reflected a small-cap bias. On average, the size factor contributed +0.62% per year. The value factor⁵ also made a substantial contribution of +0.53% per year. Exposure to lower volatility companies (negative volatility exposure) was a key factor in reducing

Table 2 Return contribution of minimum variance versus S&P 500, 1973–2012.

Factor	Annualized contribution (%)
Style factors	+1.20
Sectors	+0.50
Asset selection	–0.07
Total	1.64

Table 3 Average active exposure and return contribution for key styles, 1973–2012.

Style factors	Exposure (z-score)	Annualized contribution (%)
Size	–0.25	+0.62
Value	+0.09	+0.53
Volatility	–0.31	+0.13
Momentum	+0.03	–0.22

portfolio risk while at the same time contributing +0.13% to returns. The most serious detractor was momentum, which contributed –0.22% per year. Active sector exposures added 0.50% annually mainly due to consistent exposure to consumer staples and variable exposure to energy companies. Underweighting technology hurts the strategy over the simulation period as shown in Table 4.

Figure 2 shows how the exposure of minimum variance to size,⁶ value, volatility, and momentum

Table 4 Average active exposure and return contribution for key sectors, 1973–2012.

Sectors	Exposure (%)	Annualized contribution (%)
Consumer staples	+4.81	+0.40
Energy	+1.64	+0.34
Technology	–4.06	–0.28

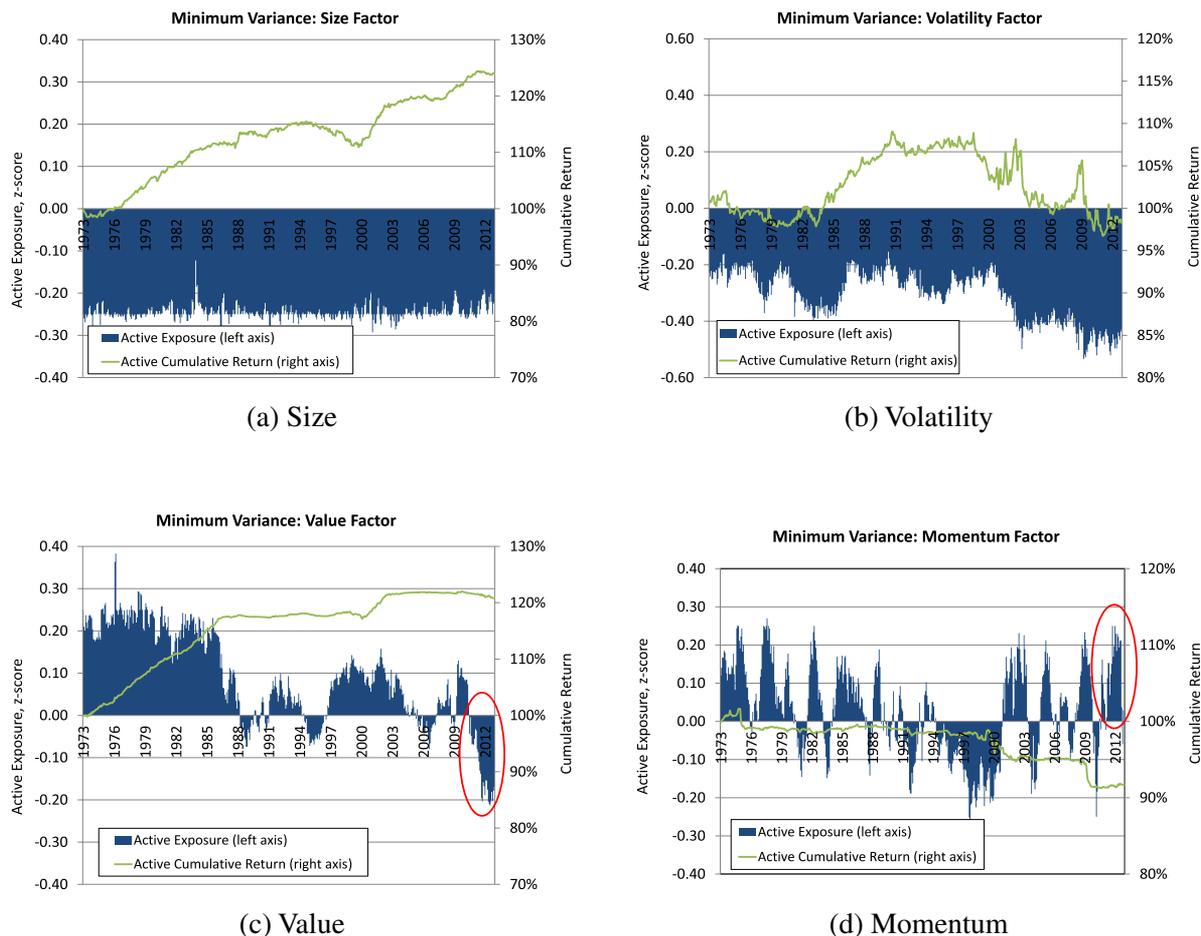


Figure 2 The relationship between minimum variance and four style factors, 1973–2012. Exposure is measured on the left axis, and contribution is measured on the right axis.

changed over time. The size and volatility exposures were relatively stable, whereas the value and momentum exposures varied dramatically over time. Although the minimum variance strategy has, on average, reflected a value bias, this tilt has reversed in the last two years. Figure 2(c) shows a positive active exposure of minimum variance to value (blue bars) coupled with a positive contribution to active return from 1973 until 1986 (green line). The contribution of value to active return was flat between 1986 and 1998 although the exposure varied. In 1998, the strategy regained its value bias, and

after the dot-com bubble burst, the bias contributed substantially to the active return until 2003. Recently, the value exposure has turned negative and has been at a historic low, as emphasized in Figure 2(c) by the part of the chart circled in red. This shift toward more expensive stocks suggests what could be a relatively high cost to entering the minimum variance strategy. Over the same period, the momentum exposure has been relatively high, as emphasized in Figure 2(d) by the part of the chart circled in red. Figure 3 shows active exposures of minimum variance to value and momentum in a single frame.

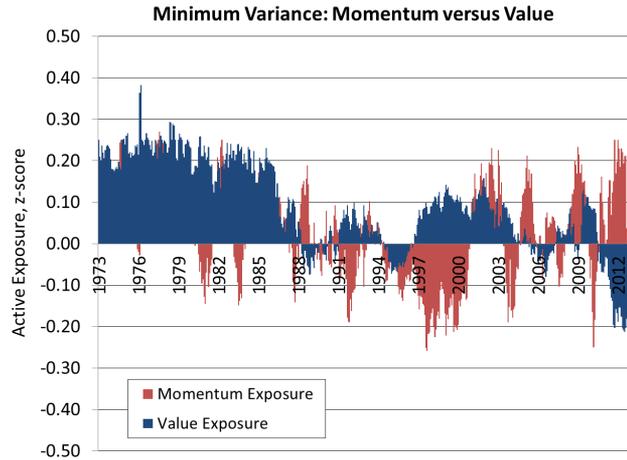


Figure 3 Active value and momentum exposures of minimum variance, 1973–2012.

3 Restoring value

Investors can restore the value tilt to a minimum variance strategy by implementing a factor constraint in the optimization process. We simulate the minimum variance strategy with a value tilt over the 1973–2012 period and analyze the impact on exposure to the value factor in Figure 4. By construction, the value-tilted version maintained a consistent positive tilt toward value and thereby ensured a cheaper cost of entry, as measured by value, to the minimum variance strategy.

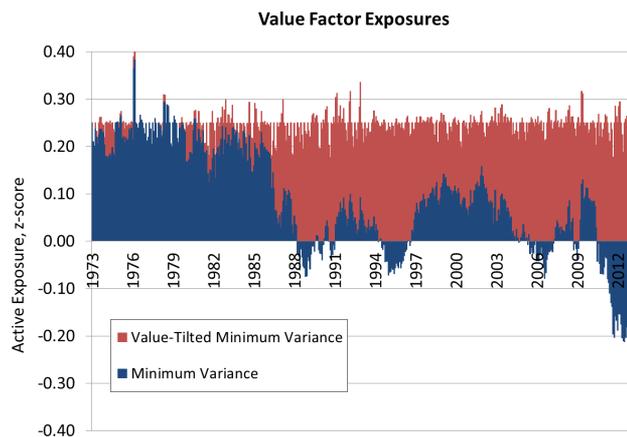


Figure 4 Active exposures of value to the minimum variance and the value-tilted minimum variance strategies over time, 1973–2012.

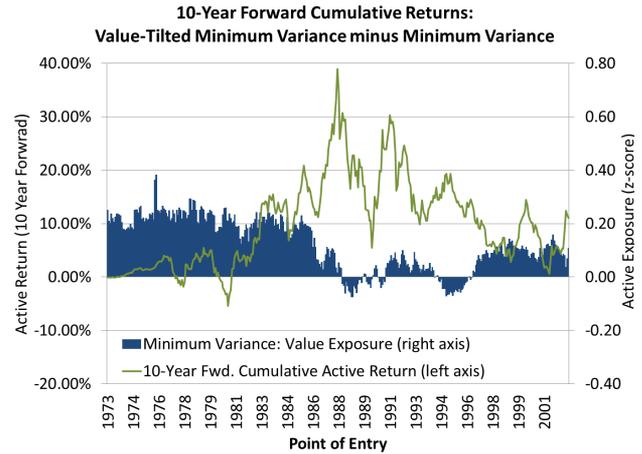


Figure 5 Difference in 10-year cumulative returns between a value-tilted minimum variance strategy and a standard minimum variance strategy at points of entry between January 1973 and December 2002 (green line). Also shown in blue is the value factor exposure of the standard minimum variance strategy.

We analyze the impact of this constraint on returns by comparing the return differences of the two strategies with rolling points of entry. Figure 5 shows the 10-year cumulative returns differences between January 1973 and December 2002, superimposed over the value factor

Table 5 Summary statistics for minimum variance and value-tilted minimum variance, 1973–2012.

Strategy	Minimum variance	Value-tilted min. variance
Annualized returns	11.44%	11.82%
Return difference vs. S&P 500	+1.64%	+2.02%
Annual standard deviation	12.89%	13.18%
Beta	0.78	0.80
Tracking error	5.17%	5.02%
Max down year	–26.46%	–26.67%
Upside capture	86.7%	88.6%
Downside capture	70.3%	71.4%
Sharpe ratio	0.50	0.51

exposure of the standard version. Since 1980, every point of entry has led to 10-year outperformance of the value-tilted minimum variance strategy (green line). This is attributable to the fact that after 1986 the minimum variance strategy has lost its strong value bias (blue bars). A strategy that incorporated a consistent value tilt resulted in slightly better performance over this period. Summary statistics that compare minimum variance and value-tilted minimum variance are presented in Table 5.

4 Conclusion

Minimum variance strategies have gained popularity in the wake of the financial crisis as chaos-shy investors have sought stability. However, while it has lower volatility than other equity-based investments, minimum variance is a complex strategy whose relationship with market benchmarks evolves over time. A factor analysis elucidates the nature of this evolution, and it shows that excess return of minimum variance over the S&P 500 between January 1973 and December 2012 can largely be attributed to implicit tilts toward stocks that have lower market capitalization and volatility, and a higher ratio of earnings to price. Recently, however, the tilt toward value has been reversed and a tilt toward momentum has emerged. This is consistent with the large inflows to the strategy.

Investors who seek more control over their investments can still pursue a minimum volatility strategy by controlling the value exposure in the optimization process. Historically, maintaining a consistent value tilt improved returns and achieved a cheaper cost of entry into the strategy. While there is never a guarantee that a particular active exposure will produce a particular set of returns in the future, constrained active exposures do provide investors with means to effectively incorporate their objectives or views,

and to correct for implicit and perhaps unintended factor tilts.

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Notes

- ¹ For more information, see *IndexUniverse*, <http://www.indexuniverse.com/sections/features/18730-top-10-etf-asset-gainers-of-2013.html>. The Powershares S&P 500 Low-Volatility Portfolio ranked second out of all 129 Powershares ETFs as of May 28, 2013, <http://invescopowershares.com/flows/>. This increase in investor interest reflects part of a broader interest in smart beta ETF strategies in general, which can be described as rules based like indexing but active in the sense of incorporating some sort of factor tilt. In the year ending March 31, 2013, inflows into smart beta ETFs rose by 45%. For details see “Large spike in smart beta investments,” by Chris Flood, *Financial Times*, June 16, 2013, <http://www.ft.com/intl/cms/s/0/d11df550-d4d0-11e2-b4d7-00144feab7de.html#axzz2WUp6U4j5>.
- ² The question of whether low-risk anomalies exist after accounting for frictions is an active area of study. See, for example, Fu (2009) and Li *et al.* (2013).
- ³ Long/short minimum variance strategies are analyzed by Alvarez *et al.* (2011).
- ⁴ At each rebalancing, the minimum variance portfolio is constrained to keep turnover less than 5% and to stay within fixed bounds of the benchmark. The maximum holding in any asset is 3%. Risk factor exposures are constrained to stay with 2.5% of the benchmark.
- ⁵ The Barra USE3 has three value-type factors: earnings yield, book to price, and dividend yield. Here, we represent value with earnings yield.
- ⁶ In the Barra USE3 model, a negative size exposure indicates a tilt toward smaller stocks.

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